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A Two-Dimensional Transverse Magnetic Propagation Model of a Sine Wave Using Mur Boundary Conditions

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Abstract

A two-dimensional (2-D) transverse magnetic formulation of a propagating sine wave source disturbance was numerically simulated using the finite difference-time domain (FD-TD) methodology. The nonreflecting boundary conditions due to Mur were used at the boundary surfaces. Electric field intensity distributions resulted over a progressive time expansion to illustrate the propagation effect over the entire 2-D mesh. The imposition of the Mur boundary algorithm produced accurate results when the second approximation was used and when the source was located reasonably far from the mesh boundary.

TABLE OF CONTENTS

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LIST OF FIGURES

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1. INTRODUCTION

The two-dimensional (2-D) finite difference formulation of time domain electromagnetic-field problems is a convenient tool for solving scattering problems. It can be easily applied to conducting obstacles as well as magnetic obstacles that can be homogeneous or inhomogeneous of arbitrary shape. When a space-time mesh is implemented and Maxwell's equations are replaced by a system of finite-difference equations on the mesh for a problem dealing with an open domain, there exists the imposition of boundary conditions for an open system (i.e., the domain in which the field has to be computed is unbounded). A methodology for limiting the domain in which the field is computed is effectuated by using a mesh of limited size, but yet large enough to contain the obstacle, and by using a boundary condition on the outer surface of the mesh, such that the unbounded surrounding is modeled as accurately as practicable. Boundary conditions of this type are called absorbing boundary conditions. Many authors have proposed methods for absorbing boundary conditions, such as Taylor (1969), Taflove and Brodwin (1975), Taflove (1980), Merewether (1971), and Kunz and Lee (1978).

However, all of these previous methods have the disadvantage of causing considerable reflections when the fields near the boundary of the mesh do not propagate in a specific direction. In this report, a potentially superior method will be implemented based upon Engquist and Majda (1977) and especially upon Mur (1981), to simulate a 2-D transient transverse magnetic (TM) distribution of electric and magnetic field intensities for a typical sine wave propagating from the center outward in all directions using the Mur absorbing boundary conditions. This analysis will demonstrate, in a limited way, how electromagnetic waves can propagate, emanating from a source of electromagnetic disturbance as typified in one of the gas turbine engine components. It is hoped that this study will serve as a starting point to look at the electromagnetic interference (EMI) produced by a starter, causing the analog electronic control unit diagnostic connector to abort the actual start of the gas turbine engine.

Frequency domain characteristics for the scattered signal can be obtained by Fourier transformation of the time-domain scattered signal. The previously mentioned problem is solved by

a FORTRAN program that can numerically compute, using the finite difference-time domain (FD-TD) method, the scattered fields of a sine wave source emanating from the center of the mesh.

Hence, the mathematical and numerical background of the FD-TD approach is reviewed for the simplifying case of TM excitation in a 2-D space. Discretization of Maxwell's equations for lossy media, stability, simulation of absorbing boundary conditions, and scattered field formulation are also discussed.

2. MATHEMATICAL AND NUMERICAL REVIEW

The FD-TD method, proposed by Yee (1966), is the direct solution of the time-dependent Maxwell curl equations. For this method, difference approximations are applied to both space and time derivatives in Maxwell's equations. By knowing the initial, boundary, and source conditions, equations are solved using the time-marching procedure. The actual wave propagations and interactions are thus simulated in numerical computations.

2.1 Difference Equations and Node Distribution. Maxwell's equations governing the propagation of electromagnetic waves in an isotropic, homogeneous medium are:

$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu} \nabla \times \vec{E} \tag{1}$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon} \left[\nabla \times \vec{H} - \sigma \vec{E} \right], \tag{2}$$

where μ, ϵ, and σ can be functions of space.

For a TM spatial lattice in a 2-D rectangular (x,y) coordinate system, equations 1 and 2 can be discretized as:

$$H_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right) = H_x^{n-\frac{1}{2}}\left(i,j+\frac{1}{2}\right) - \frac{\Delta t}{\mu}\left[\frac{E_z^n(i,j+1) - E_z^n(i,j)}{\Delta h}\right] \tag{3}$$

$$H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right) = H_y^{n-\frac{1}{2}}\left(i+\frac{1}{2},j\right) + \frac{\Delta t}{\mu}\left[\frac{E_z^n(i+1,j) - E_z^n(i,j)}{\Delta h}\right] \tag{4}$$

$$E_z^{n+1}(i,j) = \left[\frac{1-\frac{\Delta t\sigma}{2\epsilon}}{1+\frac{\Delta t\sigma}{2\epsilon}}\right] E_z^n(i,j) + \left[\frac{\frac{\Delta t}{\epsilon\Delta h}}{1+\frac{\Delta t\sigma}{2\epsilon}}\right]$$

$$\left[H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right) - H_y^{n+\frac{1}{2}}\left(i-\frac{1}{2},j\right) - H_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right) + H_x^{n+\frac{1}{2}}\left(i,j-\frac{1}{2}\right)\right], \tag{5}$$

where Δt and Δh are time and space steps, respectively.

For inhomogeneous media with continuous variation of material constants ϵ, μ, and σ, no additional work needs to be done except specifying ϵ, μ, and σ at each grid point. However, for heterogeneous media where step changes of material constants occur at interfaces of adjacent homogeneous media, some spatial treatments are necessary. Since it is assumed that no conductive media are immersed in the propagation field in this analysis, then the σ terms will not appear in the program.

2.2 <u>Stability Condition</u>. If the FD-TD method is to have a stable solution, the Courant stability condition must be satisfied:

$$V_m \Delta t \leq \frac{1}{\sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta y}\right)^2 + \left(\frac{1}{\Delta z}\right)^2}}, \tag{6}$$

where V_m is the velocity of light in the medium. When $\Delta x = \Delta y = \Delta z = \Delta h$, equation 6 is simplified to

$$V_m \Delta t \leq \frac{\Delta h}{\sqrt{3}}. \tag{7}$$

For the 2-D case, the stability condition is

$$V_m \Delta t \leq \frac{\Delta h}{\sqrt{2}}. \tag{8}$$

Note that V_m is the maximum velocity in multilayer media.

2.3 Mur Absorbing Boundary Conditions.

In this section, the finite-difference approximations of the absorbing boundary conditions are presented. These approximations have a local truncation error of the second order in all increments. The discretized form of the boundary condition for E_z at the boundary of interest, i.e., the tangential boundary for the TM case, will now be given according to Mur. The finite-difference approximation was derived using centered differences in both the space and the time increments—it has a local truncation error of the second order in all increments. The first approximation for E_z is discretized as follows:

$$E_z^{n+1}(0,j) = E_z^n(1,j) + \frac{c_0\delta t - \delta}{c_0\delta t + \delta}(E_z^{n+1}(1,j) - E_z^n(0,j)\,. \tag{9}$$

Then, a second approximation for the 2-D TM problem is discretized as

$$E_z^{n+1}(0,j) = E_z^n(1,j) + \frac{c_0\delta t - \delta}{c_0\delta t + \delta}(E_z^{n+1}(1,j) - E_z^n(0,j)) - \frac{\mu_0 c_0}{2(c_0\delta t + \delta)} \times$$

$$H_x^{n+1/2}(0,j+1/2) = H_x^{n+1/2}(0,j-1/2) + H_x^{n+1/2}(1,j+1/2) - H_x^{n+1/2}(1,j-1/2))\,, \tag{10}$$

where $c_o = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$ and where the z-dependence of the fields have been deleted from our notation since the value of z is the same in all terms. Centered differences were used for deriving (10), and these finite-difference approximations also have a local truncation error of the second order in all increments.

2.4 Sine Wave Pulse. The excitation pulse used is sinusoidal in shape. It will propagate in the +z direction and will have the representation of

$$J_z = \sin(2\pi f t), \tag{11}$$

where

$$f = 1/(50 \cdot dt)\,. \tag{12}$$

3. SUMMARY/RESULTS

Two-dimensional TM electromagnetic scattering of a sine source disturbance is numerically solved using the FD-TD method with Mur absorbing boundary conditions. Figure 1 depicts the z component of the electric field intensity distribution with respect to the x and y planes at a very early time step of the disturbance development. Figures 2–5 depict the same distributions but at progressive times to illustrate the source disturbances growth from center excitation to the outer boundaries of the 2-D mesh. The distributions depicted in this analysis show the time development of the two-dimensional wave emanating from the sinusoidal point source projected in the z-direction which was introduced as a soft source in equation (5) itself as proposed by Taflove (1980).

An analogous program can be written for the transverse-electric (TE) case that will be another followup study to this particular work. In addition, the program can also be extended to the case of three-dimensional (3-D) electromagnetic propagation from a variety of sources at different locations of the mesh.

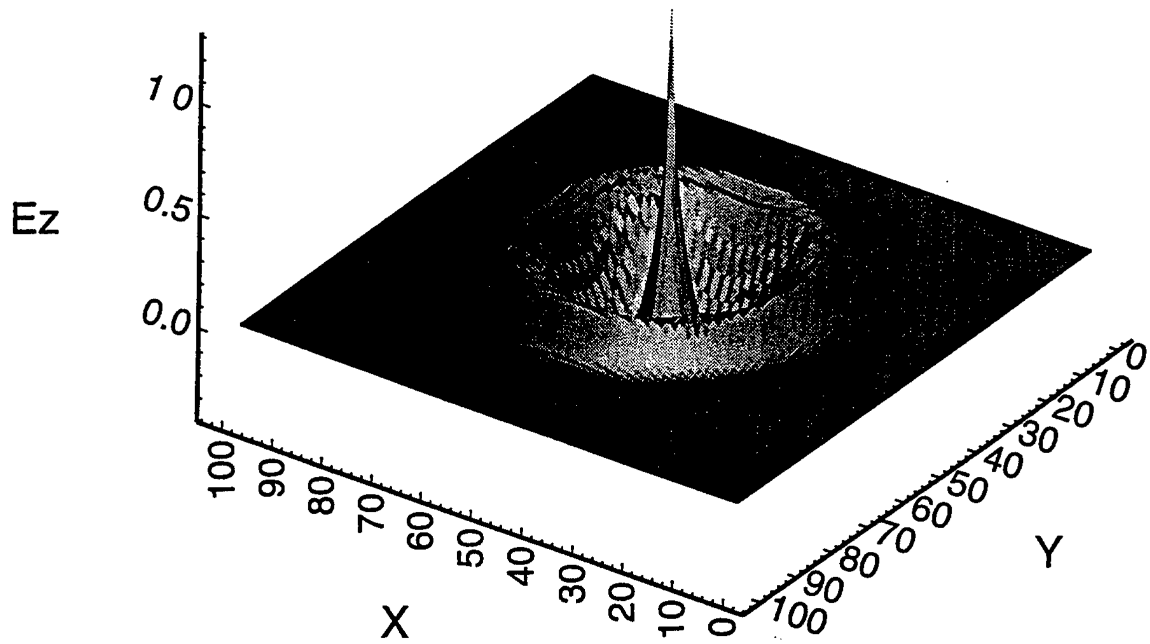

Figure 1. Distribution of electric field intensity (V/m) at time step = 50 .

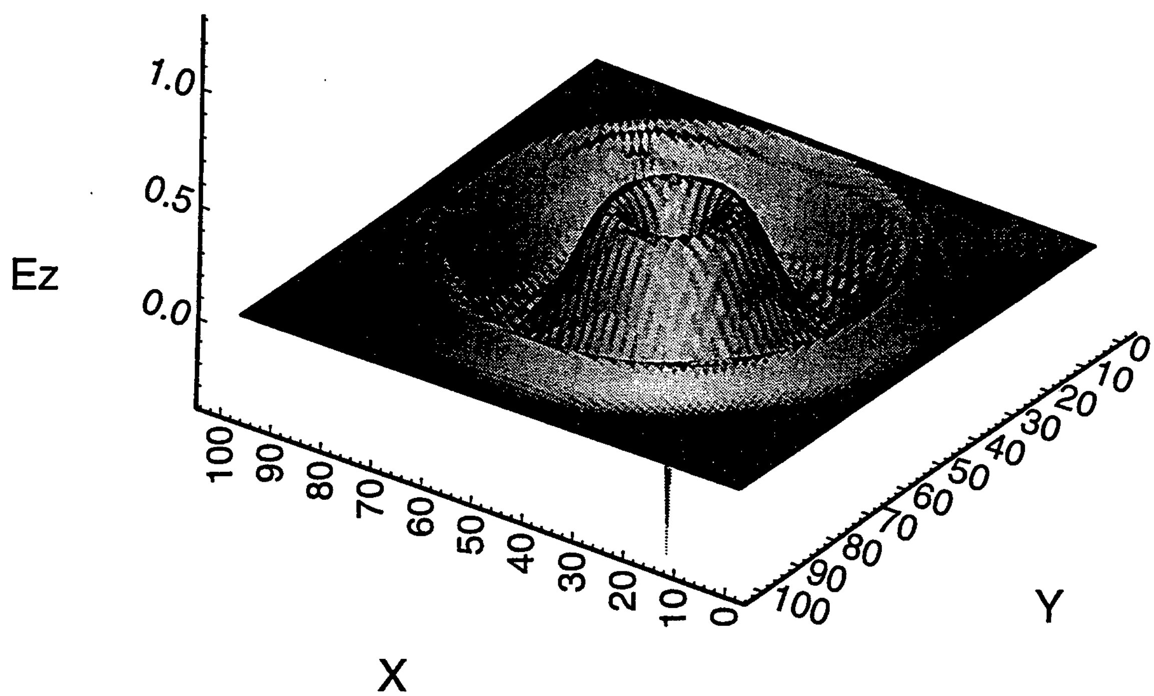

Figure 2. Distribution of electric field intensity (V/m) at time step = 75 .

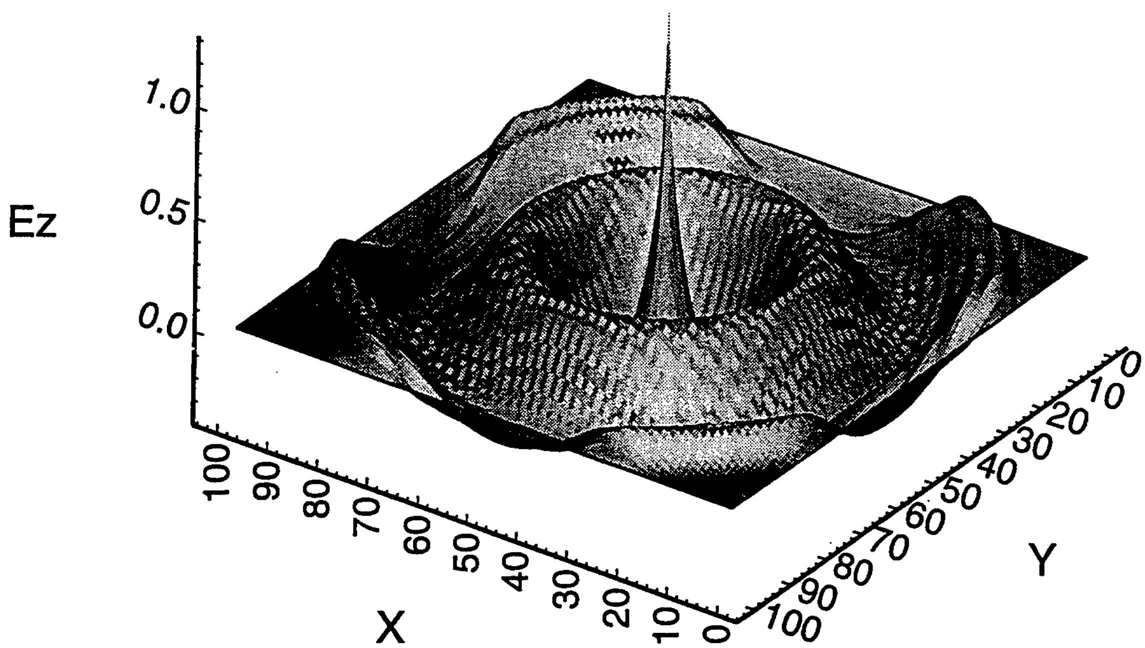

Figure 3. Distribution of electric field intensity (V/m) at time step = 100 .

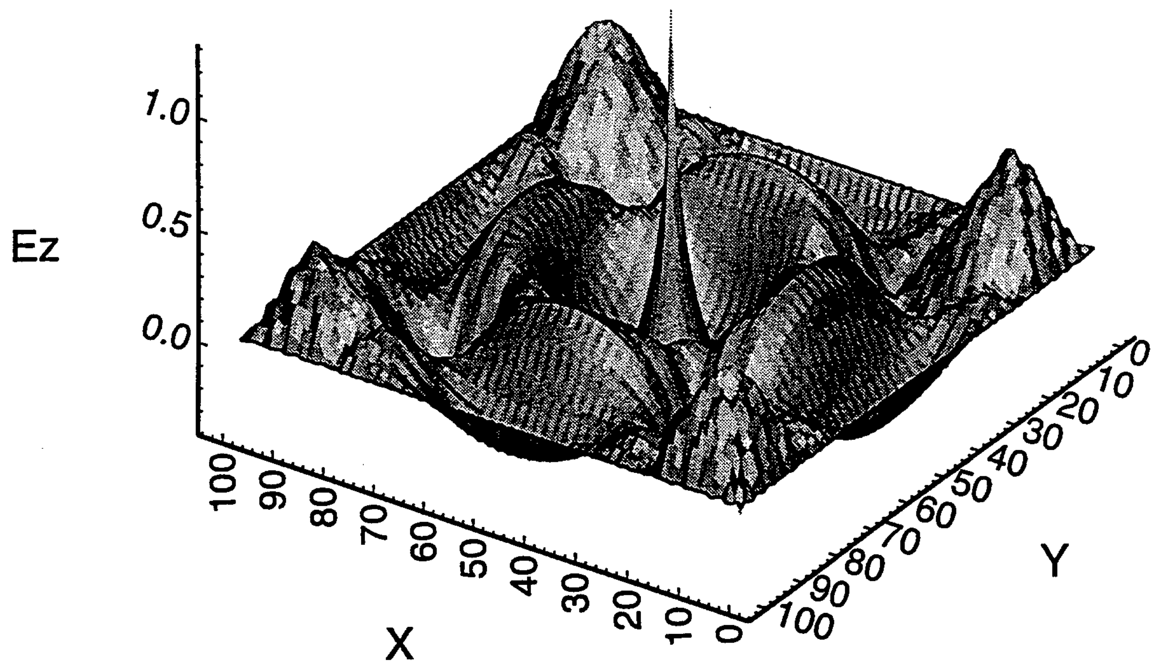

Figure 4. Distribution of electric field intensity (V/m) at time step = 150 .

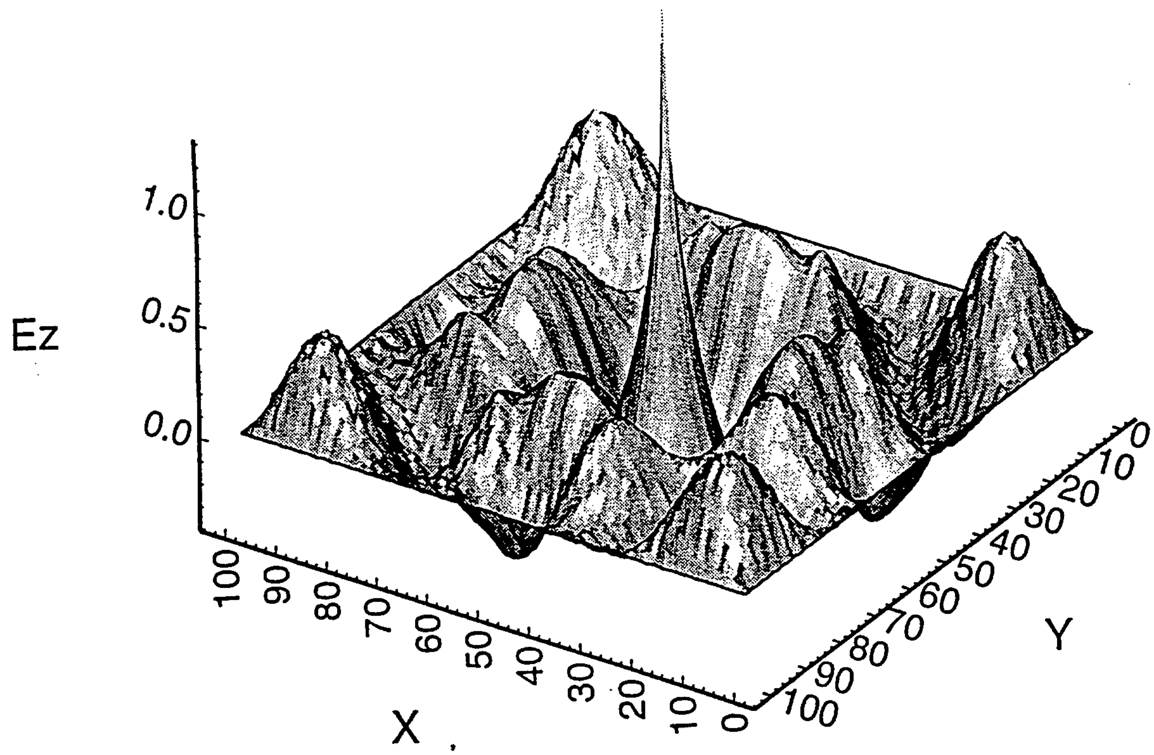

Figure 5. Distribution of electric field intensity (V/m) at time step = 200 .

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